

HACIA UNA FORMACIÓN COMUNITARIA DE PROFESORES DE MATEMÁTICAS

Towards a communal mathematics teacher education

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Resumen

Este trabajo propone una conceptualización de la formación comunitaria de profesores de matemáticas, relevante para apoyar a los docentes en la adaptación a las nuevas demandas del currículo. Basándose en una teoría enactivista de la cognición, en el trabajo se argumenta la importancia de las nociones de conciencia educativa, análisis deliberativo y subordinación de la enseñanza (de los profesores) al aprendizaje (de los profesores). Estas nociones tienen como objetivo subvertir la supuesta dicotomía entre teoría y práctica en algunas concepciones de la formación docente. Después de describir los elementos centrales de nuestro enfoque, se presentan experiencias de un curso de formación docente en Inglaterra como ejemplo. El trabajo concluye que podría ser necesario contar con una formación docente comunal si nuestro currículo pretende avanzar hacia un enfoque más socioecológico.

Palabras clave: *formación del profesorado de matemáticas; comunitaria; enactivismo; cambio curricular*

Abstract

This paper proposes a conceptualisation of a communal mathematics teacher education, as relevant to supporting teachers adapt to new curriculum demands. Drawing on an enactivist theory of cognition, the paper argues for the salience of the notions: educating awareness, deliberate analysis and subordinating teaching (of teachers) to learning (of teachers). Such notions aim to subvert the perceived binary of a theory-practice divide in some conceptualisations of teacher education. Having described core elements of our approach, experiences from one teacher education course in England are offered, by way of exemplification. The paper concludes that a communal teacher education may be needed if our curriculum is to move towards a more socio-ecological framing.

Keywords: *mathematics teacher education; communal; enactivism; curriculum change*

INTRODUCTION

In this theoretical contribution, we want to propose a conceptualisation of a communal mathematics teacher education, to help think through questions relating to how mathematics teachers can be supported in adapting to new curriculum demands. The context of this paper is our experiences working in England on a one-year postgraduate teacher education course at the University of Bristol. During the course, prospective teachers spend around 2/3rd of their time on placement in a school (where they take responsibility for teaching some classes), and 1/3rd at the university. We are conscious that the UK is unlike some other countries, in that school and university-based elements of teacher education weave together in patterns set by each institution. For example, at Bristol, prospective teachers spend an initial 3 weeks at university, then 3 weeks partly in university and partly in school, then 5 weeks just in school, 2 weeks back at university, then 10 weeks in school, 2 more weeks at university, a final 5 weeks in school and then just under 2 weeks in a final university element.

In some conceptualisations of teacher education, school-based, practicum work is viewed as “practice” and university-based work is viewed as “theory” (Burn et al., 2022). Having separated out practice and theory, questions then emerge as to their relationship (e.g., uni-directional or bi-directional). It is commonly found, for example, at the beginning of a teacher education course in England, that prospective teachers act in the classroom in spontaneous ways which are ineffective (Brown & Coles, 2011). At the end of the course, for most of them, they act in the classroom in spontaneous ways which are effective. It seems a valid question to ask what role “theory” has in such development.

To foreshadow the overall argument, we will be suggesting that the separation of theory from practice leads to conceptual tangles that cannot be unravelled. Instead, from an enactivist position, ways of working within mathematics teacher education (“educating awareness”, “deliberate analysis” and “subordination”, which we will elaborate below) capture how prospective teachers can learn to become effective, in a manner that is sensitive to their needs, particularly during a time of curriculum change, without a separation of theory from practice. We will then argue that, together, these practices constitute a “communal” approach to teacher education and that such an idea is relevant to adaptations needed in face of impending socio-ecological crisis. In considering teacher education in this writing, we have prospective teachers in mind, however, we believe the ideas extend to work with in-service teachers.

In the next section we consider briefly debates around conceptualising teacher learning, which have influenced thinking in England. We then set out the enactivist theory of cognition, which implies a quite different way of thinking about teacher learning, before exemplifying these ideas with examples of ways of working, drawn from the University of Bristol.

CONCEPTUALISING TEACHER EDUCATION

Until relatively recently, in England, the majority of secondary mathematics teachers qualified to become teachers through taking a “Postgraduate Certificate of Education” (PGCE), which is a one-year course. To gain entry onto a PGCE course requires a Bachelor’s degree, in mathematics, or a mathematics related subject. There are “Subject Knowledge Enhancement” courses available (up to 28 weeks long) allowing someone with a degree not related to mathematics (but who usually studied and succeeded at mathematics to 18) to work on their subject knowledge and take up a PGCE place to teach mathematics. The PGCE course includes subject specific and general pedagogical components (however, the distinction between didactics and pedagogy is not one that is often used, or well-known, in the UK). The balance of these varies (in Bristol around 2/3rds of university time is subject based, 1/3rd general). We note there are also Bachelor’s degrees which lead to a teaching qualification. These are more common at primary than secondary level in England. There are also one-year primary PGCE routes, for those with a degree not relating to the study of education.

In the last five years a variety of different routes into teaching have emerged, but the detail is not of concern to the argument in this paper. PGCE courses were historically based at universities, usually in Schools or Departments of Education. The staff in those Departments were free to set up their courses in whatever way they wanted (with the caveats that the proportion of time in school was set at 2/3rds and that there were inspections every 4 or so years, when the standard of provision was graded and made public). The result of this context was a relatively rich diversity of approaches, in some cases drawing theoretically on researchers from that institution.

For example, at the University of Cambridge, the primary PGCE centred around the knowledge quartet (Rowland, et al., 2005), whose authors taught on the course. At the University of Birmingham, there was a strong influence on the secondary mathematics PGCE from the Science of Education (Gattegno, 2010), driven by staff who had been influenced by working personally with Gattegno (we will be drawing on Gattegno’s ideas later in this paper). The Institute of Education, in

London, was known for its “critical” approach to primary and secondary teacher education (White & White, 2022). At the University of Bristol (where we work) a key influence on the secondary PGCE was Stenhouse (1991) and the notion of teachers as researchers (into their own practice); the course for secondary mathematics teachers evolved (influenced by enactivism) in ways we will set out in the next section.

We want to focus here on the University of Oxford, since their conceptualisation is one that both has been re-thought recently (Burn et al., 2022) and, also, offers a contrast (conceptually) to the approach we will be proposing. McIntyre (1995) wrote about the notion of “practical theorising” as the guiding idea behind the Oxford PGCE course. Practical theorising suggests there is a bi-directional influence between theory and practice, in learning to teach. McIntyre argued that what prospective teachers need to do is to take ideas from university (theory) and take what they observe in schools (practice) and, in their own work, find a way of bringing them together. And this work is the work of practical theorising. When these ideas were introduced, teachers in school were seconded to work on the teacher education course and university staff seconded to work in schools. As can be seen in a recent publication (Burn et al., 2022), practical theorising continues to be an idea that drives the practice of teacher educators at Oxford.

Hirst (1990) published a critique of the notion of practical theorising. Two arguments were: (a) that engaging in practical theorising was simply too hard to expect the majority of prospective teachers to be able to do; and (b) that there was a danger of relativism, as there are no established criteria by which to judge competing theories. There is not space to delve into these arguments, but what we want to point out is that Hirst’s critique agrees with McIntyre (1995) in the separation of theory and practice. In other words, both theorists take this separation as unquestioned.

One of the reasons for wanting to bring into question the separation of theory from practice is that such a framing seems go along with reports of tensions between school-based and university-based elements of teacher education courses (Todd et al., 2022). Anecdotally, it is a relatively common refrain among teacher educators in England to report on such tensions and a sense that universities can be seen to be advocating for classroom practices (such as group work and discovery) which are impractical to implement in schools. We want to work through whether there might be alternative conceptualisations and whether such ideas might support a teacher education which was not experienced as in tension with work in schools. In order to discuss an alternative, there is a need to set out some of the underlying enactivist theory of cognition and this is the focus of the next section.

AN ENACTIVIST THEORY OF COGNITION

The School of Education at the University of Bristol has been referred to as the “Bristol School” of enactivism in mathematics education (Reid, 2014) and enactivist ideas (Varela et al., 1993) have influenced the design and teaching of the secondary mathematics PGCE course. One key idea is that “knowing is doing, doing is knowing” (Maturana & Varela, 1992, p. 29), in other words, knowing is not associated purely with self-conscious or propositional statements. There is knowing in any unconscious skilful behaviour. In fact, enactivism views unconscious and effective behaviours as a better model for thinking about the nature of cognition in general than more self-conscious, linguistic reflections.

The notion of effective action is made quite precise by Maturana and Varela (1992, p. 29) as meaning to operate in a manner which sustains relationships within a domain of living beings. If my actions allow key relationships to persist, they are effective. Brown and Coles (2000) discuss the complex decision-making required of a teacher. The action in a classroom happens too quickly for deliberation to be of much use. Those authors combined enactivist insights with the proposal of Damasio (2006) that, as humans, we develop somatic markers of our past decisions and actions. Decisions with positive outcomes accrue positive somatic markers and those with negative

outcomes accrue negative somatic markers. According to Damasio, those somatic markers determine what arises in any moment, as a possible action, thereby simplifying the decision-making process. What is not effective does not even arise as a possibility.

We want to recognise that there are now several strands of enactivist thinking. We have drawn on some of its older roots and pointed to just three key ideas (knowing as doing, effective action, somatic markers) out of a great many we could have chosen. For reasons of space and coherence, we have chosen to elaborate just those concepts we use to help think about teacher education.

In terms of those concepts, the issue we are grappling with in this paper could be framed as, how can effective action at university support effective action in school? We very deliberately are not viewing what is learnt in university as theory and what is learnt in school as practice. From an enactivist perspective, the question is, what does a university course need to do, so that what sustains relationships in a university setting (i.e., what makes for effective action) helps to sustain relationships in a school setting? In both settings we conceive of the work as crossing any imagined theory-practice divide.

Having set out, in quite theoretical terms, an enactivist view of cognition and learning, in the next sections we describe three related notions, which draw on enactivism and the writings of Gattegno (2010), whose ideas we elaborate below, and which we take as coherent with enactivism. These notions are: educating awareness; deliberate analysis; and subordinating the teaching of teachers to the learning of teachers. We offer these three sets of ideas in a spirit of inquiry – seeking what resonance they may have with others. All three relate to the non-separation of theory and practice. The notion of educating awareness is, for us, a way of conceptualising learning. Deliberate analysis is a key mechanism for learning (not separating theory and practice) and subordination relates to our roles as teacher educators in setting up contexts for learning where theory and practice are not separated.

Educating awareness

This section draws on the writings of Gattegno (2010). Although Gattegno would not have used the descriptor of being an enactivist, his work has strong links, through his focus on dynamics and actions in learning mathematics. Gattegno (1998) offered insights into human cognition and developed curriculum materials. He had a focus on visible and tangible approaches to learning and, in one of his most famous aphorisms, suggested that *only awareness is educable*. Mason (2002) extended the aphorism to: “only awareness is educable, only emotion is harnessable, only behaviour is trainable” (p.141). It is not straight-forward to pin down what Gattegno meant by awareness. It is worth noting here that Gattegno collaborated with Piaget in the 1950s and was a translator (from French to English) of some of Piaget’s writing. “La prise de conscience” is a phrase used by Piaget that can be translated as awareness, and also as “realisation”. Mason offers a definition (distilled from Gattegno’s writing) of awareness as “that which enables action” (2011, p.12). The non-separation of thought and action or knowing and doing (or theory and practice) is evident in these formulations.

In terms of our concerns in this article, we are interested in how the framing of educating awareness might help us conceptualise our work as mathematics teacher educators, and in particular, in supporting teachers navigate new curriculum demands and the different agenda of university and school settings. If we think about learning to teach in terms of developing more and more layers of awareness about the classroom and about mathematics, then a key question is whether and how, what prospective teachers are becoming aware of in university links to what they are becoming aware of in school. When we, as teacher educators, hear reports about prospective teachers finding university and school messages conflict (e.g., “my university is advocating particular forms of pedagogy that are unrealistic in my school”) then this says to us that the aims, in terms of awareness, at both university and school, are similar. In other words, for there to be a conflict or

tension across school and university settings, then the agenda of both places need to be about the same kind of thing (e.g., approaches to teaching particular topics).

Our own approach to this potential dilemma, at the University of Bristol, is that the “content” of university sessions is at a meta-level to the “content” of likely school advice and guidance. What we mean by this is that university sessions are *about* school experiences, i.e., at university, prospective teachers are supported in developing awareness *about* learning to teach and at school they are engaged in the direct process of learning to teach. Or another way of framing this is that in schools, prospective teachers develop awarenesses about teaching and at university they develop awarenesses about awarenesses about teaching. What is significant for us in framing the task of teacher education in the way we have done here, is that teacher education practices can then be relatively stable even through curriculum changes (Brown et al., 2021) – a point we return to in the conclusion.

Deliberate analysis

A second key idea for us is the enactivist notion of *deliberate analysis* (Varela, 1999). Varela discusses deliberate analysis in the context of how experts learn. Varela characterises the behaviour of experts and novices as involving spontaneous actions, undertaking without deliberation. The difference, however, is that for the expert, those spontaneous actions can be reflected on and deconstructed, to get back to the “intelligent awareness” which led to that action, whereas, typically for the novice, it is not possible to access the reasons for particular actions. The link between deliberate analysis and educating awareness is apparent from this definition.

In Brown and Coles (2011), the authors describe how deliberate analysis can be used in the process of learning to become a mathematics teacher – as a way of learning from the very start of a PGCE course. In other words, engaging in deliberate analysis is a way for novices to learn like experts. A key aspect of the use of deliberate analysis is that of “staying with the detail”. We view one feature of university sessions on a PGCE course as developing with prospective teachers the skills to engage in deliberate analysis about their own teaching.

In Brown et al. (2021) there was a description of the use of the heuristic *experiences to issues to actions* which captures the way in which deliberate analysis is operationalised through our work as teacher educators. We engage prospective teachers in successive cycles of analysis, involving acting; reflecting on action to identify issues; reflecting on issues to identify strategies; acting ... and so on. Within these cycles it is crucial that the reflection on action begins with the detail of experience and that the identifying of strategies gets to “action targets”, i.e., things which can be done/tried by the prospective teacher. The role of somatic markers appears here. We use emotional markers of events to help prospective teachers locate those moments from lessons they have taught that hold the potential for new learning. A negative somatic marker will point to an issue where new actions are needed. A positive somatic marker will point to an issue where it may be helpful to articulate the actions which were taken and which could be repeated.

Subordinating teaching (of teachers) to learning (of teachers)

The last of our three elaborations returns us to the work of Gattegno. Gattegno (2010) proposed that his teaching approach implied *the subordination of teaching to learning*. What we interpret this to mean is that the progress of lessons needs to be led by the needs and responses of those learning, not by the agenda of those teaching. Gattegno wrote about the need to stay in continual contact with what learners are learning and he developed the use of materials such as Cuisenaire rods, that allow learners to be active and their responses to tasks visible to the teacher.

A key element of the subordination of teaching to learning is also that teachers make use of learners’ powers of the mind. The powers were summarised (Coles, 2011, p. 50) as:

- the power of extraction - finding “what is common among so large a range of variations” (Gattegno, 2010, p. 10)
- the power to make transformations – e.g., my Dad ~ my Mum’s partner ~ my sister’s Dad ~ my uncle’s brother
- handling abstractions – e.g., any noun is a label for a general set of objects
- stressing and ignoring – e.g., focusing on one aspect of perception to the exclusion of others.

Gattegno’s approach to teaching mathematics was to design a curriculum which allowed learners to use those powers – and which expected learners to use those powers – in becoming confident in doing mathematics. As far as we are aware, Gattegno’s ideas of subordination and powers of the mind have not, to date, been written about in relation to learning to teach.

What the subordination of teaching to learning means (to us) in the context of teacher education, relates to the ideas expressed in the Educating Awareness section, namely, that the content of university sessions is the teaching of prospective teachers. In other words, as teacher educators, subordinating our teaching (of teaching) to the learning (of teachers) necessitates that we provide tasks that allow the focus to be on the learning of the teachers about teaching. In the next section, we elaborate on what some of these tasks might look like and how they allow teachers to engage the powers of their minds.

We note here that while subordination implies we do not have one model of teaching, which we want prospective teachers to adopt, it does imply that we have conviction in our model of teacher education. We are not arguing against having models of teaching (and indeed like to work with prospective teachers so they have conviction about their own models). But our model of teacher education is at a meta-level to any model of teaching in schools.

EXAMPLES FROM A MATHEMATICS TEACHER EDUCATION COURSE

In this section, we offer some details about practices on our teacher education course, influenced by the three ideas set out above. In conclusion, we will be pointing to the “communal” nature of each practice and why this may be significant. We have chosen three practices to describe and analyse, which are all examples from the first few weeks of the course, as we try to set up ways of working with our prospective teachers. We begin each one with a description (indented) of the practice and then we reflect on the practice in light of the ideas set out above.

Mini-teaching

In the first week of the PGCE course, prospective teachers have to teach each other something non-mathematical for 10 minutes. We usually split the prospective teachers (usually 30 in a cohort) into two groups and they do their mini-teaching just to that group. In a letter they receive before the course, they are told to prepare for this “mini-teaching”.

A teacher educator organises the mini-teaching session, which includes ensuring that precisely 10 minutes are taken. If prospective teachers finish before the 10 minutes, they need to fill the time and, the teacher educator will cut off any activity in mid-flow if needed, when the time gets to 10 minutes. Immediately following this teaching, the rest of the group are then invited to say what they learnt and anything that got in the way. They are asked to engage in a professional dialogue – potentially saying things that might appear negative but offering them from their own experience. Having heard the feedback, the last word is with the person who gave the mini-teaching, to reflect on how it went compared to their plan.

The key idea of the mini-teaching is to give prospective teachers an experience, at the very start of the course, of teaching in a safe environment. As teacher educators we want to set up a cycle of teaching, getting feedback, identifying “action-targets” (i.e., things to do next time) and then

teaching again. The mini-teaching allows us to set up this cycle. The feedback offered by the group supports a form of deliberate analysis, as the person doing the teaching is able to reflect on decisions made and their impact. Examples of such feedback are: “when you stood in front of the board, I could not see what was on the screen”. Other issues that can arise are, for example, that handing out resources can take up 4 or 5 of the 10 minutes, leaving little time for any activity. As teacher educators, we have no control over what issues are discussed or what feedback is given. The mini-teaching is also, for us, an important moment in establishing ways of working together and a professional dialogue, that may include challenge as well as support.

Three-way discussions

After each time of being in school, when prospective teachers return to the university, the first session we run with them is a three-way discussion. A teacher educator will arrange desks so that prospective teachers are sitting in groups of 3 (some 4s if needed). The 3s are invited to act as “reflective teams” for each other. In turn, one person has “time”, to share experiences they have had in school. The teacher educator will usually invite everyone to think back over their time in school and locate any moments of discomfort, or comfort, any moments that stay with them. When it is a person’s “time” then the other 2 in the group aim to listen actively, not thinking about what to say next, not offering solutions or similar experiences, but rather helping the person elaborate on their experience and get to new action-targets. After each person has had their “time” (usually 10 to 15 minutes each) the teacher educator then invites a group to share an issue they discussed which still feels unresolved. These issues are then discussed as a whole group, where we share a range of strategies for addressing the issue. An example might be an issue being raised of how to deal with not knowing the prior knowledge of students. Strategies suggested by the group might include, for example, doing a diagnostic assessment, speaking to their teacher from last year, looking at student records, having a range of questions for students to try at the start.

As teacher educators we are often struck by the power of the three-way discussions, which we sense from the energetic nature of the conversations taking place. The sessions are examples of subordinating teaching to learning, as we cannot know what issues will arise. Again, our focus as teacher educators is on supporting prospective teachers to get to new awarenesses about their teaching. We invite them to use their power of stressing and ignoring, to isolate specific moments from their experiences in school. We make use of emotion, or affect, in the belief that moments which retain some emotional hue (which may relate to somatic markers), indicate something that can usefully be worked on further. These are likely to be either moments of discomfort where there is still something unresolved which can be discussed, or something relating to how that prospective teacher would like to be in a classroom and where, this time, actions taken that could be repeated can be discussed.

From our perspective, as teacher educators, we take the process of identifying “issues” and then planning potential “strategies” to address those issues, as acts which cut across theory-practice divides. Our prospective teachers do not appear to experience a disjunction between school and university; university sessions are *about* school and support reflection on school experiences.

Working on mathematics

In the first week of the course, prospective teachers engage in a series of “workshops” with a mathematics focus. In one workshop, prospective teachers spend 4 minutes working in pairs on 15 different tasks. They move around the room so that they work with a different partner on each task. The tasks were originally aimed at students of age 14 (being based on a workshop created by prospective teachers many years ago). The tasks have changed over time, and now aim at appealing to different images of mathematics, i.e., some tasks are pure number or algebra tasks, some involve estimation, some involve technology. After each task, prospective teachers are invited to note down

what they liked or did not like, what they thought the task was assessing, and any reflections on working as a pair.

We are conscious, in an English context, that many people coming on to the course speak in interview, about enjoying the solitary nature of doing mathematics. We are therefore keen to set up expectations about working together. We want prospective teachers to be curious about each other's approaches to the subject and to supplement their own ways of working with alternatives. After the 15 workshop tasks have been done by each person, we form a large circle of chairs to discuss and de-brief on the experience. We want prospective teachers, at this stage, to hear how what one person likes, another one does not (this always happens in reflection) and to begin the process of opening themselves to alternative images of what mathematics is about. We invite reflections about working together also, and typically there will be comments that some people have found this hard and prefer to work alone. We simply note to the prospective teachers, that working together on mathematics is a skill they need to learn over the year and is something they will be asked to do repeatedly – and so we invite a reflection on developing awareness about what strategies have made this successful.

TOWARDS A COMMUNAL MATHEMATICS TEACHER EDUCATION

We have offered, so far, a conceptualisation of teacher education as involving processes of: educating awareness, deliberate analysis and subordinating teaching (of teachers) to learning (of teachers). In exemplifying these processes with three practices on the teacher education course at the University of Bristol, we have pointed to how teacher education need not be caught in dilemmas about the connection between theory and practice. If university sessions are *about* school experiences, they can support the processes of developing awareness of teaching in school, and prospective teachers need not experience a sense of disjuncture between the two settings. Indeed, we propose the notion of educating awareness as one that escapes the practice-theory dichotomy, or the knowledge-action divide. Our perspective of enactivism refuses such dualisms, equating knowing and doing.

In looking across what we have offered so far, we are struck by how the practices we are proposing, and that we have exemplified, seem to move away from an individualistic or compartmentalised approach to mathematics teacher education. In the “mini-teaching”, as prospective teachers reflect back to each other what they learnt and what got in the way, the process of learning to teach becomes distributed across the group. They become entangled in each other's' journeys of learning to teach. We each (including teacher educators) have a responsibility to share our own experience, allowing the “teacher” to reflect on the implications of their actions (an example of deliberate analysis). Similarly, with the “three-way discussions”, prospective teachers are involved in each other's' developing awareness and again support processes of deliberate analysis through their listening and probing. The reflecting team and wider group play a crucial role in developing plans for next actions in each other's' teaching. And, in working on mathematics and setting up paired work, we begin a process of prospective teachers developing what might be new ways of being, in the doing of mathematics.

Scheiner and Bower (2023), in offering a critique of metaphors of unpacking mathematics, point to the colonial force behind processes of individualisation and compartmentalisation, in our education systems. We view the separation of theory from practice (or university knowledge from school knowledge, or knowledge from action) as one example of such compartmentalisation. In proposing the idea of a communal mathematics teacher education, we are wanting to offer ways of working that act against assumptions such as the individualised nature of knowing. In the discussions of teaching issues that we observe on the PGCE course (e.g., during three-way discussions) there is a palpable sense of learning to teach being a group endeavour.

The recent ICMI Study on Curriculum Reform (Shimizu & Vithal, 2023) highlighted the vital importance of adequate teacher training and support, for any new initiative, particularly when this initiative involves technology. There are growing calls for a move to a more socio-ecological conception of mathematics teaching (Coles et al., 2022). The socio-ecological captures ideas such as not taking nature as a fixed background for research concerns and moving away from viewing the individual as the unit of learning. The notion of a communal approach to teacher education is, therefore, particularly relevant to such a socio-ecological framing.

Scholars working towards the inclusion of the socio-ecological in mathematics education (e.g., see the ICMI Symposium on Mathematics Education and the Socio-Ecological, from 20th March 2023) are convinced that curriculum change is coming. And, indeed, not linked to socio-ecological ideas, many countries are already engaging in curriculum reform (as noted in the ICMI Study). The images we have tried to offer of a teacher education at a meta-level to the concerns of prospective teachers offers one response of how teacher education can adapt to changing curricula. In other words, in the face of new curriculum demands, practices supporting educating awareness, deliberate analysis and subordination, may be able to stay relative steady, since the content is provided by the teachers and their experience of any new curriculum.

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